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Cosmological Models with a Specified Trajectory on the Energy Phase Plane

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Abstract—We present a method of cosmological model classification according to their thermodynamic properties, on the basis of their energy phase trajectories. It is shown that the basic elements of the classification are the properties of energy phase trajectories at the beginning and end points of the evolution. Using the proposed method, we have analyzed some types of cosmological models which are important subject to the modern views on the evolution of the observed Universe. In particular, we have analyzed cyclic and quasi-cyclic models used as an element in slow rolling theory.

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1. INTRODUCTION

Refs. [1, 2] studied a two-component cosmological model with a spatially flat Friedmann-Robertson-Walker (FRW) metric and matter in the form of a scalar field and a perfect fluid with a general-type variable equation of state $p = \gamma(t)\varepsilon$. Models of this kind quite naturally contain the basic features of the modern experimental data on the Universe evolution history. In particular, these models almost always contain primary inflation and secondary accelerated expansion at the last stage of the evolution. In the models considered in [1, 2], it was supposed that the components of matter are always in thermal equilibrium. Due to this condition, the equilibrium temperature of the Universe is zero when it is born, then it rapidly (exponentially) grows at the primary inflationary stage and rapidly falls down after its end. This is different from the classical viewpoint that the Universe was born in the Big Bang with infinite temperature [3]. On the other hand, the models suggested can also be considered with violated thermal equilibrium, which widens the opportunities of their application. However, before performing such studies, it makes sense to find a general classification of all kinds of models from a thermodynamic viewpoint. It is this problem that is solved in the present paper.

We will perform an analysis of a class of models on the basis of the assumption that the cosmological dynamics is governed by the evolution of the scalar field, determined by its inherent properties, e.g., the quantum ones. Although our study is based on a classical (non-quantum) approach, the basic properties of the field, including the quantum ones, may be formally accounted for using the characteristics of its energy evolution. This approach has been named in [2] the *scalar field governed models*. Here we will also adhere to this terminology. The aim of this paper is to construct a method for an analysis and classification of cosmological models using their thermodynamic properties, starting with an equation that determines the evolution of the total energy of the governing field. This problem is solved in the first part of the paper. The second part contains an analysis of some models with properties of interest, which illustrate the opportunities of the suggested scheme which studies the models on the basis of energy characteristics of the matter components and their time evolution.

2. THE “THERMODYNAMIC” REPRESENTATION OF THE COSMOLOGICAL DYNAMIC EQUATIONS

Our starting point will be the standard Einstein equations for the spatially flat FRW metric with a self-interacting scalar field and a perfect fluid,

$$H^2 = \frac{\kappa}{3}\left(\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] + \varepsilon\right), \qquad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{d}{d\phi}V(\phi). \qquad (2)$$

Here, $H = \dot{R}/R$ is the Hubble parameter, R is the scale factor, κ is Einstein's gravitational constant, ε is the fluid energy density, and $V(\phi)$ is the self-interaction potential of the scalar field ϕ. The first equation is an Einstein equation as such while the second one is the field equation for ϕ. This set of equation is a starting point for the majority of cosmological scenarios in the framework of the FRW metric [3]. Since the model contains two matter components, the field and the fluid, the above equations should be supplemented with an equation for the fluid pressure p:

$$p = -\frac{1}{\kappa}\left(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right) - \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad (3)$$

which is, generally speaking, a consequence of the first two equations. This equation makes it possible, using rather simple transformations, to bring the original equations to a more convenient form (see [1, 4]):

$$\mathcal{P} = -W - \frac{1}{\sqrt{3\kappa}}\frac{\dot{W}}{\sqrt{W+\varepsilon}}, \qquad (4)$$

$$p = -\varepsilon - \frac{1}{\sqrt{3\kappa}}\frac{\dot{\varepsilon}}{\sqrt{W+\varepsilon}}, \qquad (5)$$

$$R = R_0 \exp\left\{\sqrt{\kappa/3}\int\sqrt{W+\varepsilon}dt\right\}, \qquad (6)$$

where the total energy W and the effective field pressure $\mathcal{P}$ have the standard form

$$W = \frac{1}{2}(\dot{\phi})^2 + V(\phi), \quad \mathcal{P} = \frac{1}{2}(\dot{\phi})^2 - V(\phi). \qquad (7)$$

To analyze the set of equations (4)–(6), it is useful to introduce the function

$$U(\phi) = \dot{\phi},$$

which connects the scalar field and fluid parameters [1, 4]:

$$\sqrt{3\kappa}U\sqrt{W(\phi)+\varepsilon} = -W'. \qquad (8)$$

The full set of equations (4)–(6) allows one to calculate the cosmological model dynamics from specified thermodynamic parameters of the scalar field and the fluid. Therefore this set of equations may be called the "thermodynamic" representation of the cosmological dynamic equations. For the scalar field, the "thermodynamic" information is the information on its self-interaction potential $V(\phi)$ or on its total energy $W(\phi)$, as has been shown in [4]. Since Eqs. (4)–(6) contain a time derivative of W, whereas the functional form of $W(\phi)$ so far cannot be found confidently enough from some physical considerations, it is more convenient for the analysis to fix the field properties by indicating the functional dependence $W(t)$. In [1, 2], as a version for determining the total field energy evolution an a subsequent calculation of the function $W(\phi)$, an equation was used that has the following general form:

$$\dot{W} = -Q(W). \qquad (9)$$

An autonomous nature of this equation means that the field W evolves by its intrinsic laws, unrelated to the evolution of other matter components in the Universe. For examples, such a field can spontaneously decay due to its inner quantum processes. If there is no other matter, the Universe evolution is determined solely by the decay law of this field which is a solution to Eq. (9). Choosing different kinds of the function $Q = Q(W)$ in this equation, one can obtain different models characterizing the basic properties of the whole Universe dynamics. If there are other forms of matter in the Universe, there should be energy exchange between the field and these other forms of matter, which should lead to a thermal equilibrium between them [1]. In this case, due to the autonomous nature of (9), the Universe dynamics will also be determined solely by the field evolution law $W = W(t)$. Therefore such an approach was named in [2] the "field governed models."

The properties of the second matter component, the perfect fluid, will be described, by analogy with [1, 2], by an equation of state of the following form:

$$p = \gamma(t)\varepsilon. \qquad (10)$$

As shown in [1, 2], this form of the equation of state makes it possible to describe the changing percentage of separate perfect fluid components in their common mixture, each of them creating its own partial pressure.

3. THE THERMODYNAMIC INTEGRALS OF MOTION

To complete the thermodynamic description of the cosmological dynamics based on Eqs. (4)–(6), it is necessary to supplement them with relations describing the evolution of matter temperature. To this end, using the second law of thermodynamics for reversible processes, we obtain the general relation

$$\varepsilon = -p + T\frac{\partial p}{\partial T}, \qquad (11)$$

where T is the fluid temperature. Then we obtain the following expression for the fluid entropy density, in which the equation of state (10) has already been taken into account:

$$\sigma = \frac{\varepsilon + p}{T} = (1+\gamma)\frac{\varepsilon}{T}. \qquad (12)$$

Using the equations of motion, the latter equation is brought to the form of the following conservation law:

$$R^3(1+\gamma)\frac{\varepsilon}{T} \equiv R^3\sigma = s_0 = \text{const}. \qquad (13)$$

Here, $s_0 > 0$ is the entropy of a comoving fluid volume. We will call this well-known relation *the first thermodynamic integral of motion*. Unifying this relation with (5), we arrive at its useful modification which looks as follows:

$$T = -\frac{R^3}{s_0}\frac{\dot{\varepsilon}}{\sqrt{3\kappa}\sqrt{W+\varepsilon}}, \qquad (14)$$

The second integral of motion is obtained in a similar form if one formally introduces the "effective temperature" of the field Θ and the field entropy density $\mathcal{S}$ with the aid of the two relations

$$W = -\mathcal{P} + \Theta\frac{\partial \mathcal{P}}{\partial \Theta}, \quad \Theta d\mathcal{S} = dW + \mathcal{P}dV. \qquad (15)$$

In this case, the second thermodynamic integral of motion can be written as follows:

$$R^3\mathcal{S} = \mathcal{S}_0 = \text{const}. \qquad (16)$$

This law implies an expression for Θ similar to (14):

$$\Theta = -\frac{1}{\mathcal{S}_0}R^3\frac{\dot{W}}{\sqrt{3æ}\sqrt{W+\varepsilon}}. \qquad (17)$$

Here $\mathcal{S}_0$ is the the conserved thermodynamic entropy of the field ϕ in a comoving volume.

The parameter Θ, under the condition $\Theta > 0$, provided by the two conditions $\mathcal{S}_0 > 0$ and $\dot{W} < 0$, may be considered as a certain effective field temperature, which is connected in a natural way with some "effective equilibrium thermodynamics" of the field by the relations (15). It is therefore natural to consider the question of a thermodynamic equilibrium between the components in each elementary spatial volume, which is equivalent to the condition

$$T = \Theta. \qquad (18)$$

This relation immediately leads to

$$\frac{\dot{W}}{\mathcal{S}_0} = \frac{\dot{\varepsilon}}{s_0},$$

or

$$\varepsilon = \frac{s_0}{\mathcal{S}_0}W + \varepsilon_0. \qquad (19)$$

Let us notice that, in a functional sense, a violation of the thermodynamic equilibrium condition (18) in the form

$$\Theta = \eta T,$$

where $0 < \eta < 1$ is a constant, changes actually noting. Introduction of such a parameter is equivalent to changing the relation $q = s_0/\mathcal{S}_0$, which is equivalent to some model in equilibrium but at other values of the conserved entropies. Changes in the model dynamics will only appear if the parameter η is essentially time-dependent. However, this kind of violation of the thermodynamic equilibrium should be necessarily considered in the framework of inhomogeneous cosmological models, which may be of interest for perturbation theory, for example, in the epochs of phase transitions like the recombination epoch.

To complete the general description of the thermodynamics of models under consideration, we present an expression for the parameter $\gamma(t)$:

$$\delta = \gamma + 1 = -\frac{1}{\sqrt{3\kappa}}\frac{\dot{\varepsilon}}{\varepsilon\sqrt{W+\varepsilon}}$$
$$= -\frac{q}{\sqrt{3\kappa}(qW+\varepsilon_0)}\frac{\dot{W}}{\sqrt{(q+1)W+\varepsilon_0}}. \qquad (20)$$

Here $q = s_0/\mathcal{S}_0$.

4. THE ENERGY PHASE PLANE $(W, -\dot{W})$

The whole set of relations for the "thermodynamic" description of the field-governed Universe evolution now makes it possible to formulate a sufficiently full classification of all possible models on the basis of studying the single equation (9). It is connected with the fact that most of the known spatially flat cosmological models may be reduced in one or another way to a choice of the function $Q(W)$. The simplicity and autonomous nature of Eq. (9) allows for analyzing the properties of phase trajectories of this equation on the (W, Q) plane and connecting them with the nature of the cosmological dynamics, i.e., the changes of the scale factor $R = R(t)$ and the temperature $T = T(t)$. A useful parameter in the analysis of various models with the aid of phase trajectories is the full evolution time between given points on the phase plane. This quantity is determined from Eq. (9) itself and has the form

$$\mathcal{T} = -\int_{W_0}^{W_1}\frac{dW}{Q(W)}. \qquad (21)$$

An infinite evolution time is obtained in two situations:

1) $Q(W_0) = 0$, or $Q(W_1) = 0$;

2) $W_0 = \infty$.

To begin with, consider the case that there is only one matter component. For certainty, let us suppose that this component is the field. Then, on the basis of the general properties of trajectories on the phase plane, one can state that all admissible trajectories of cosmological dynamics on the (W, Q) plane can begin at any point $P_0(W_0, Q_0)$ of its first quarter and end at a point with $P_1(W_1, Q_1)$ with coordinates satisfying the condition

$$0 \le W_1 \le W_0.$$

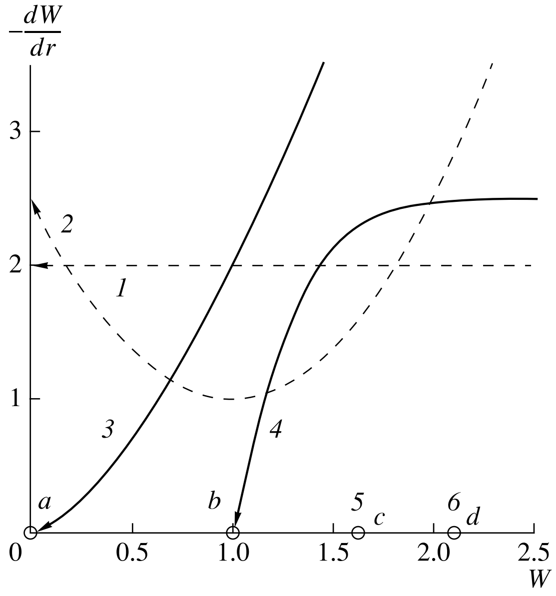

Fig. 1. Admissible energy phase trajectories of cosmological models. Models 1 and 2 are those with suddenly terminated evolution, 3 is a Friedmann model, 4 is an asymptotically de Sitter model, 5 and 6 are de Sitter models; a, b, c, d are stationary points of Eq. (9).

This follows from the thermodynamic requirements: $\Theta \geq 0$, $\mathcal{S}_0 > 0$, $W \geq 0$, $\dot{W} \leq 0$, and the dominant energy condition for the model. For the one-dimensional dynamic system (9), only stationary points $\dot{W}|_P = 0$, situated at the abscissa axis $Q = 0$, can be attractors.

Formally, the trajectories may terminate on the ordinate axis $W = 0$ of this plane. Such points are, however, connected with a singular behavior of solutions to Eq. (9). Such an example is the model with $Q(W) = Q_0 = \text{const}$, whose trajectory is depicted in Fig. 1 and denoted by 1. For such a model, $W = W_0 - Q_0 t$ on the finite segment $0 < t < W_0/Q_0$. After its substitution to Eq. (6) we obtain

$$R(t) = R_0 \exp\Big\{ -k\Big(W_0 - Q_0 t\Big)^{3/2}\Big\}.$$

For such models, a solution only exists in finite time intervals and cannot be continued beyond the time instant at which the ordinate axis is reached, certainly except for the point $(0, 0)$ which can be an attractor. Models similar to this one (in Fig. 1, the trajectory of one of them is denoted by the digit 2) are hardly of interest from the viewpoint of cosmological dynamics. Finiteness of the Universe existence time, without an opportunity to continue it beyond a finite time interval, within which solutions of (9) are correctly defined, is an irremovable shortcoming of such models. We will call them models with a suddenly terminated evolution. Therefore it remains reasonable to analyze only models whose trajectories tend to one of the attractors on the positive part of the $Q = 0$, in particular, as $t \to \infty$.

Of interest is also a time-reversed analysis of the attractors, i.e., an analysis of the repellers as $t \to 0$ or $t \to -\infty$. It is an asymptotic analysis of the starting points of the Universe evolution. Changing t for $-t$ in Eq. (9), we obtain that, in this limit, points located on the axis $Q = 0$ can also be attractors. However, unlike the case of $t \to \infty$, among the attractors in the reversed time limit can be an infinitely remote point in the first quarter of the phase plane. Therefore the phase trajectories of admissible models can begin either at an infinitely remote point in the first quarter or on the axis $Q = 0$ and terminate on the axis $Q = 0$ only.

In principle, when analyzing the system trajectories on the energy phase plane, such trajectories are admitted whose part lies in the quarter of the phase plane where $\dot{W} > 0$, $W > 0$. Such trajectories do not contradict any of the dynamic equations but they contradict the condition $T > 0$ in the case of a thermodynamic equilibrium between the field and fluid fluctuations. This follows from the relation (14) that determines the fluid temperature. In the case of thermal equilibrium, the sign of $\dot{\varepsilon}$ coincides with the sign of $\dot{W}$, which leads in the domain $\dot{W} > 0$ to negative fluid temperatures. Let us note that a negative value of Θ can be in principle justified by the fact that this quantity has been obtained from formal relations and is only connected with the conservation law of the quantity $\mathcal{S}_0$. However, the fluid temperature is a fundamentally positive quantity. Therefore one can assume that if models with parts of a trajectory having $\dot{W} > 0$ are possible for the field, then, as the point $\dot{W} = 0$ is approached, the thermodynamic equilibrium in the system must be violated. Consequently, the model should contain some indications of the nature of thermodynamic equilibrium violation, which should provide the validity of the condition $T > 0$.

The basic dynamic properties of the phase trajectories are related to the physical properties of the cosmological models. Above all, it is easy to establish the meaning of stationary points of Eq. (9) for $W > 0$ (the points b, c, d in Fig. 1). First, the stationary points themselves represent cosmologies with a fixed matter density value, invariable in time (models 5 and 6 in Fig. 1, corresponding to the stationary points c and d). This property is inherent to de Sitter models with an exponential expansion of the Universe, underlying the inflationary models. Second, the models whose trajectories asymptotically approach stationary points with $W > 0$, possess an asymptotically de Sitter behavior, i.e., they will expand asymptotically exponentially if the trajectory slope at such a point is nonzero. The form of the function $Q(W)$ near a stationary point $W = W_1$ can be presented as follows:

$$Q(W) \sim k(W_1 - W)^{\alpha},$$
$$W \to W_1, \quad t \to \infty. \qquad (22)$$

If the slope is zero, i.e., $\alpha \geq 1$, then the asymptotic expansion will have other characteristics, which have been considered in [1] and [2]. For $1 < \alpha < 3/2$, the asymptotic expansion will be accelerated, while $\alpha \geq 3/2$ it will be decelerated. In both cases, from any point of the trajectory, the stationary point will be reached in infinite time. An exponential expansion corresponds to $\alpha = 1$. Such a model has been considered in [2]. Lastly, if the slope of the trajectory at the stationary point is equal to 90°, which corresponds in (23) to the condition $0 < \alpha < 1$, then, from any point of the trajectory which is not infinitely remote and does not lie on the abscissa axis, this point is reached in finite time. For such models, the Universe expansion will be hyper-inflationary, i.e., the Universe expands to infinity in finite time. Another type of such models are cyclic or quasi-cyclic models. Their trajectories are continued to the lower quarter of the semi=plane $W > 0$, where motion is possible only from left to right. Such models will be considered below.

Similarly to the asymptotic behavior as $t \to \infty$, one considers the reversed-time behavior:

$$Q(W) \sim k(W - W_0)^{\beta}, \quad W \to W_0. \qquad (23)$$

In this case, there are two variants of different nature. Let us denote by $\mathcal{T}_0$ the time for which the Universe, starting from an initial state with the energy W_0, reaches a state with an energy $w < W_1$, smaller than the energy of the evolution end point. This quantity can be calculated using the relation

$$\mathcal{T}_0(W) = \left| \lim_{z \to W_0} \int\limits_z^w \frac{dW}{Q(W)} \right|. \qquad (24)$$

The first variant of the evolution corresponds to a situation with $\mathcal{T}_0(W) < \infty$ and the second one to $\mathcal{T}_0(W) = \infty$. In the first case, a cosmological singularity always lies at a finite time interval from any observation instant, and at infinite time in the second case. As in the analysis of reaching the evolution end point W_1, the time $\mathcal{T}_0(W)$ will be finite, which corresponds to a finite existence time of the Universe under the condition $0 < \beta < 1$. In particular, such a situation corresponds to cyclic or quasi-cyclic models.

5. A MODEL WITH FINITE ENERGY DENSITY AND INFINITE EVOLUTION TIME

Among models with admissible energy trajectories, distinguished are models whose starting point is located on the abscissa axis, as well as their end point. This means that the energy density in the Universe is finite at its starting instant. An example of such a model is the one whose phase trajectory is specified by the relations

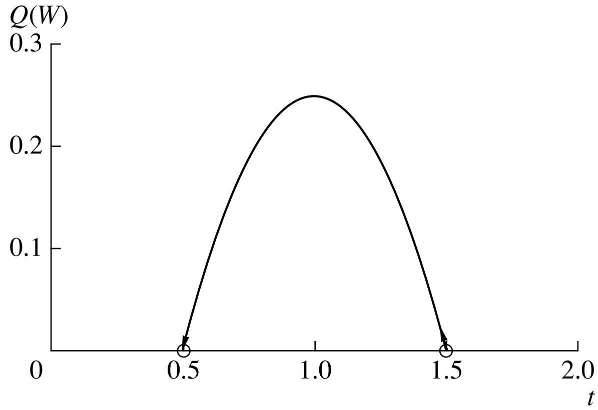

Fig. 2. Trajectory of the system (25).

$$\dot{W} = -\frac{k}{W_0 - W_1}(W_0 - W)(W - W_1),$$
$$W = \frac{W_0 - W_1}{1 + ge^{kt}} + W_1, \qquad (25)$$

and is presented in Fig. 2.

Since the slopes of the trajectory at its starting and end points are finite, these points are reached in infinite times. As $t \to -\infty$, the energy density of the system is equal to W_0 while as $t \to \infty$, it is $W_1 < W_0$. The evolution of the parameter $\gamma(t)$ and the temperature $T(t)$ are presented in Figs. 3a and 3b for some values of the parameter k that specifies the velocity of motion along the trajectory.

The most significant feature of this type of models is the impossibility to show a point in the past where the radius of the Universe had been zero. Let, for example, $R(t_0) = R_0 > 0$. Then formally, as $t \to -\infty$, $R(t) \to 0$. It is, however, impossible to satisfy the explicit condition that $\lim_{t \to -\infty} R(t) = 0$. From a mathematical viewpoint, the point $t = -\infty$ is a limiting point of the range of $R(t)$. Therefore such models can be called *models with an inaccessible cosmological singularity.*

6. CYCLIC MODELS

One more interesting class of models for which the starting and end points of the trajectory lie on the axis $\dot{W} = 0$, are those whose trajectory makes an angle of 90° with the abscissa axis at the beginning and end.

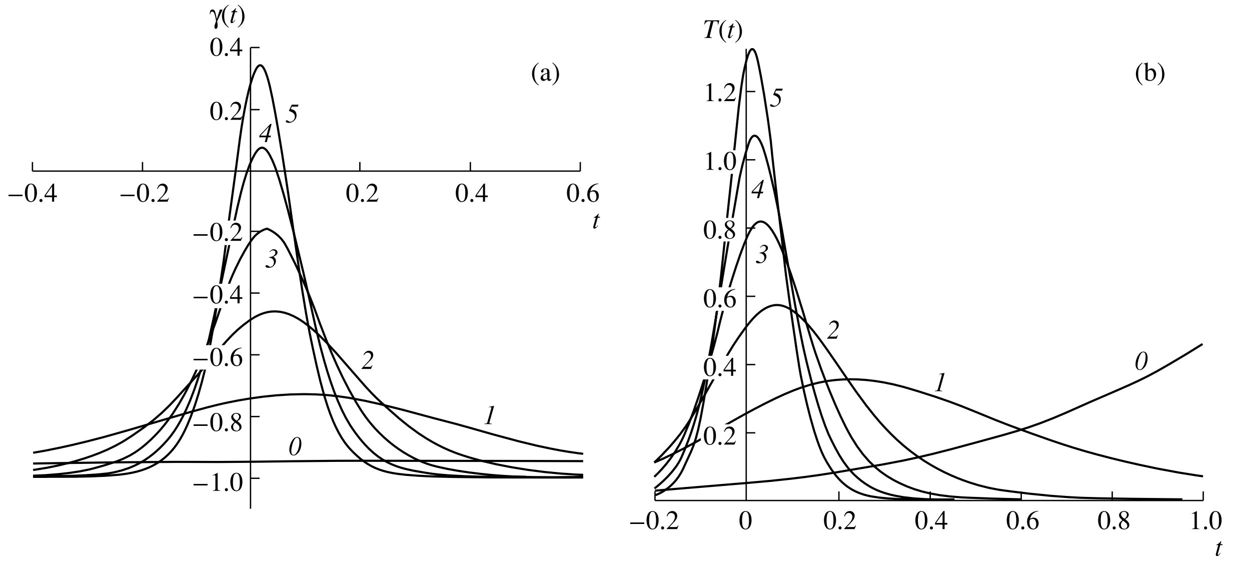

Fig. 3. The parameter $\gamma(t)$ (a) and the temperature T (b) for the model (25) at different values of the parameter k: 0—$k = 1$; 1—$k = 5$; 2—$k = 10$; 3—$k = 15$; 4—$k = 20$; 5—$k = 25$.

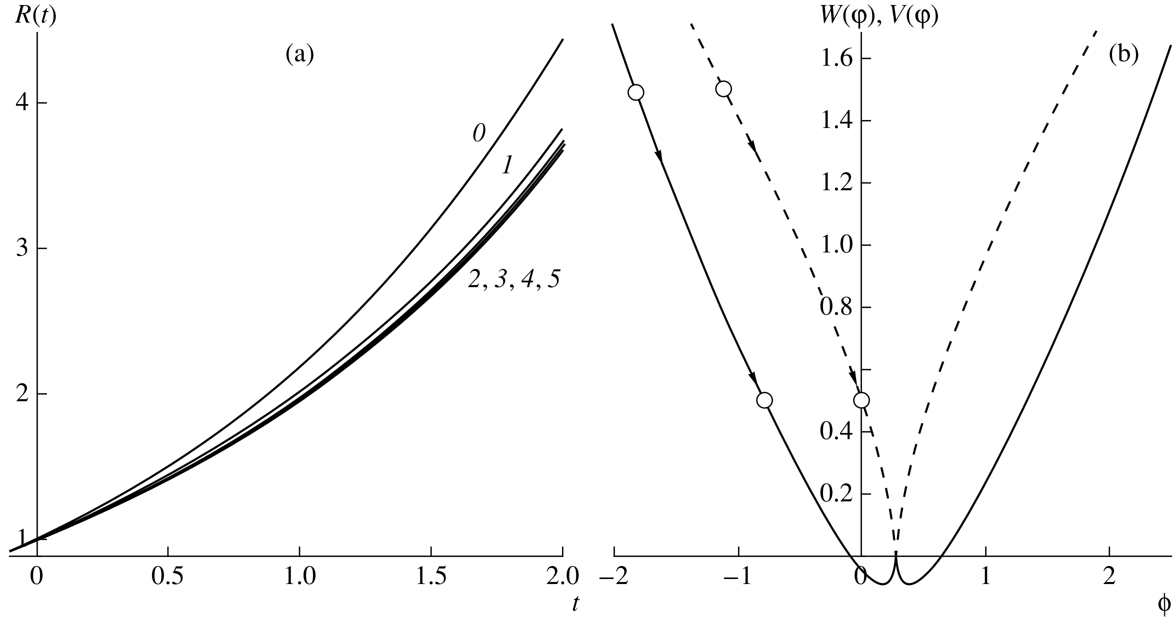

Fig. 4. The scale factor $R(t)$ (a), the total energy $W(\phi)$ (dashed curve (b)) and the self-interaction potential $V(\phi)$ (solid curve (b)) for the model (25). The numbering of curves in (a) is the same as in Fig. 3.

This means that, at the initial instant t_0 and the final instant t_1, one has

$$|dQ(W)/dW|_{t=t_0} = |dQ(W)/dW|_{t=t_1} = \infty.$$

In this case, the time of reaching the starting and end points is finite, and such models admit a smooth extension to the domain of positive values of $\dot{W}$. It means that such trajectories may be of cyclic or quasi-cyclic nature. An example of such a behavior is the model whose energy trajectory is given by the equation

$$Q(W) = \omega\sqrt{W_1^2 - (W - W_0)^2}. \qquad (26)$$

This equation is easily solved, leading to the following evolution laws for the field energy density $W(t)$ and its derivative $\dot{W}$:

$$W(t) = W_0 - W_1 \sin(\omega t),$$
$$\dot{W} = -W_1\omega\cos(\omega t). \qquad (27)$$

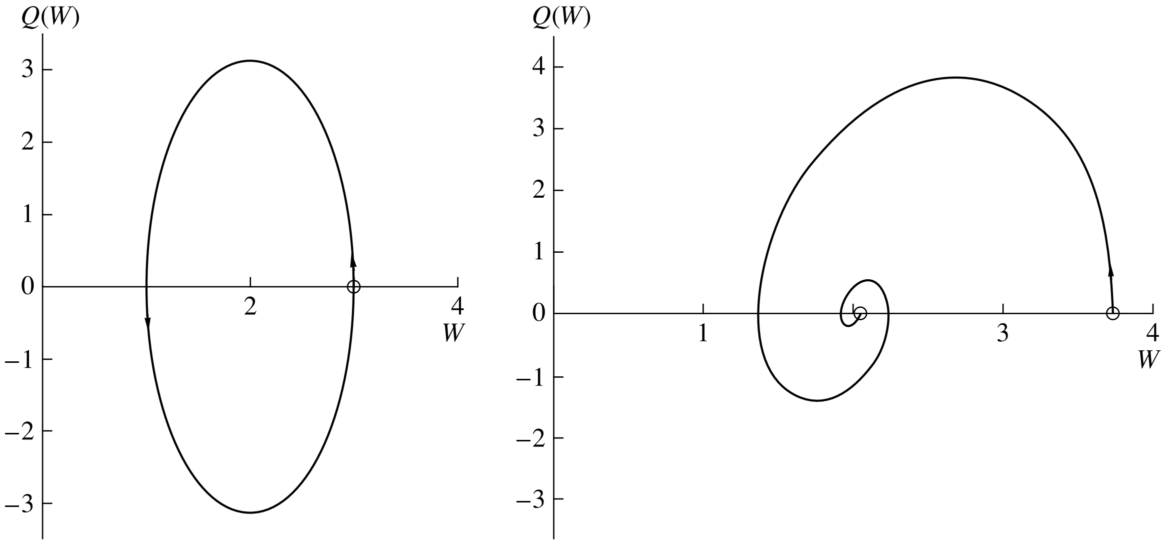

Fig. 5. Phase energy trajectories of cyclic (a) and quasi-cyclic (b) models.

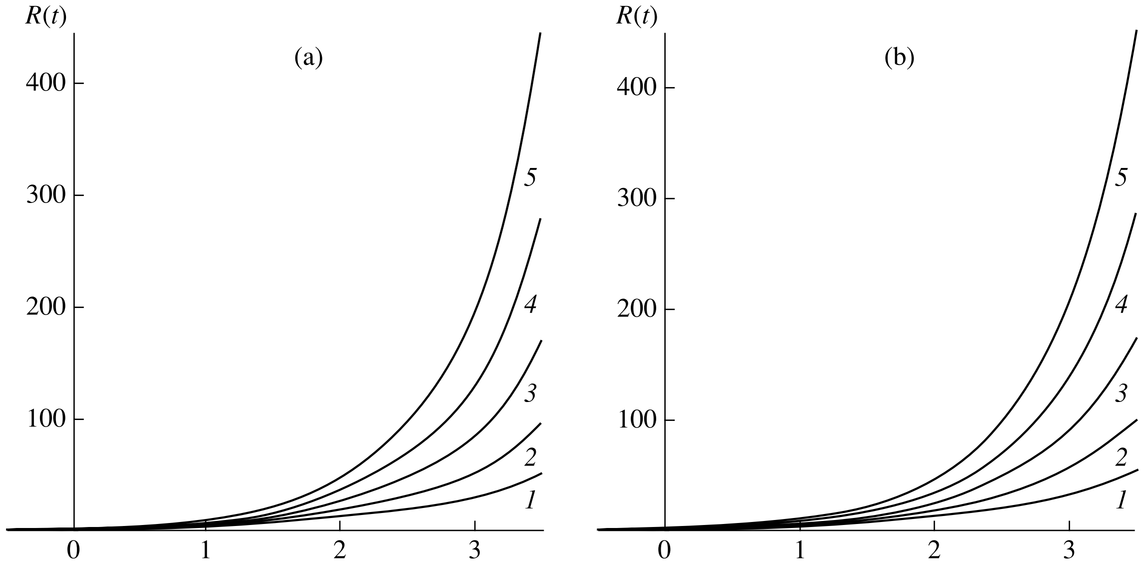

Fig. 6. Evolution of the scale factor $R(t)$ in cyclic (a) and quasi-cyclic (b) models for different values of the parameter ε_0: 1—$\varepsilon_0 = 0.1$, 2—$\varepsilon_0 = 0.5$, 3—$\varepsilon_0 = 1$, 4—$\varepsilon_0 = 2$, 5—$\varepsilon_0 = 4$.

As is seen from these relations, the energy changes periodically with the frequency ω, reaching in finite time its maximum (W_0) and minimum (W_1) values. The energy phase trajectory of this system is presented in Fig. 5a.

A model closely related to this one is another one in which the energy evolution law is given by

$$W(t) = W_0 - W_1 e^{-\lambda t}\sin(\omega t),$$
$$\dot{W} = -W_1\omega e^{-\lambda t}\cos(\omega t) + W_1\lambda e^{-\lambda t}\sin(\omega t). \quad (28)$$

The phase energy trajectory of such a model is presented in Fig. 5b. A model of this kind is a standard way to interpret the behavior of the Universe in the framework of slow-rolling models [3]. In slow-rolling models it is supposed that, after exit from the primary inflation, the field self-interaction potential tends to its minimum and performs decaying oscillations near it. It is this kind of behavior that corresponds to the model (28). Fig. 6 shows the behavior of the scale factor for cyclic models (27) and (28) in an interval of the order of two periods of the main frequency.

In both cases the plotted scale factors are monotonically growing functions, almost without difference from each other. Fig. 7 shows plots of the parameter $\gamma(t)$ for both types of models. As has already

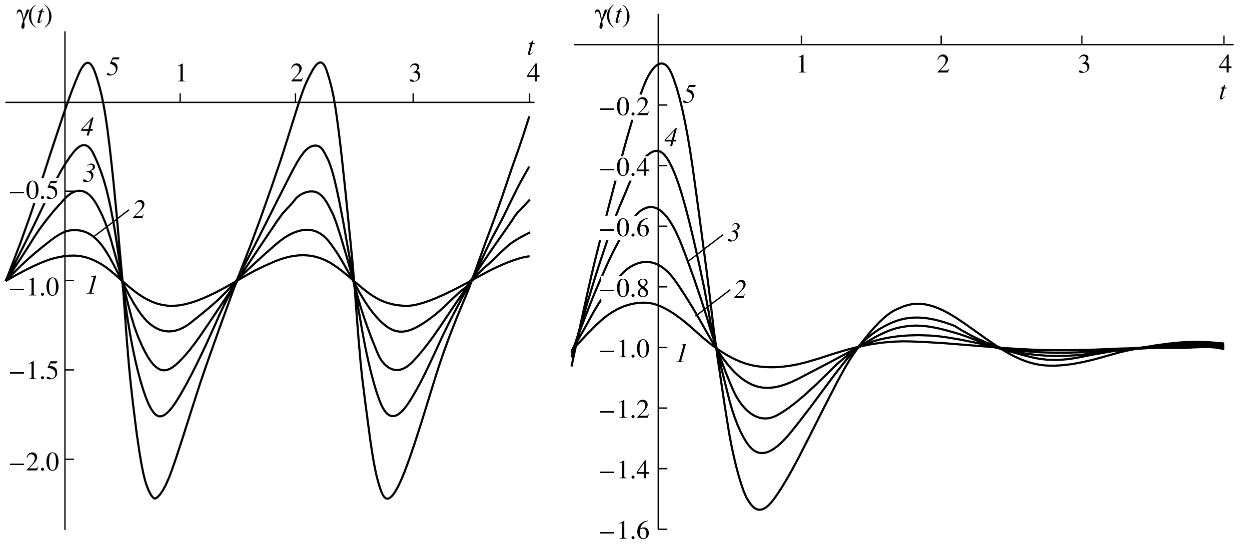

Fig. 7. Evolution of the parameter $\gamma(t)$ in cyclic (a) and quasi-cyclic (b) models for the same values of the parameter ε_0 as in Fig. 6.

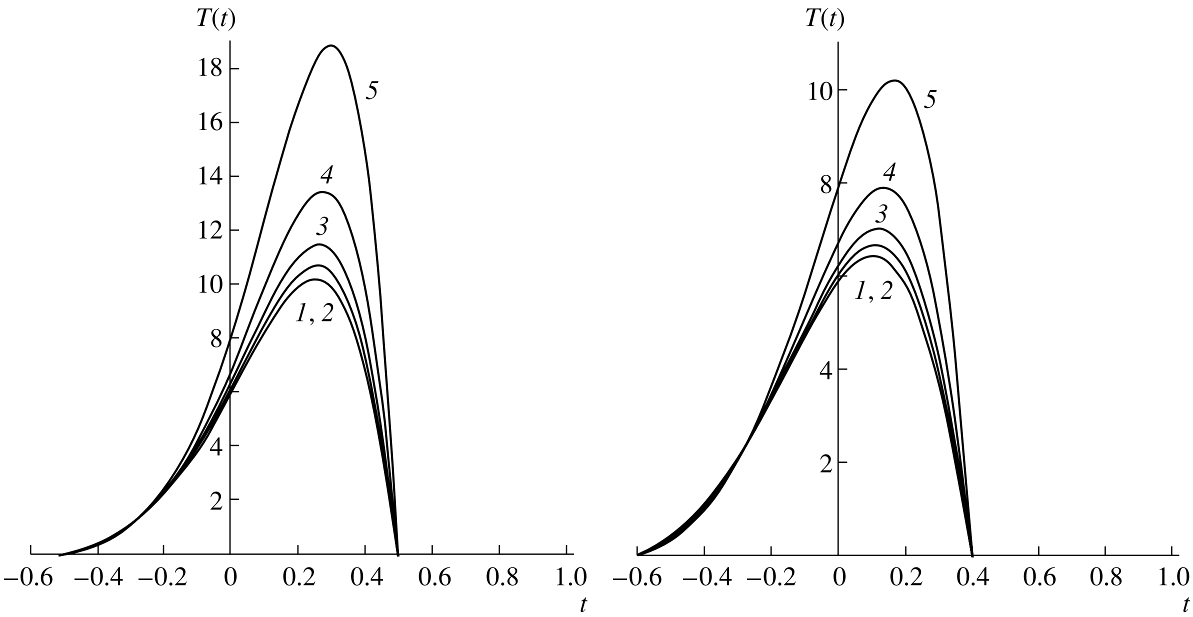

Fig. 8. Temperature changes in the cyclic (a) and quasi-cyclic (b) models for the same values of the parameter ε_0 as in Fig. 6.

been pointed out, the models whose trajectories have segments with $\dot{W} > 0$, lead to negative fluid temperatures, although it looks admissible from a dynamic viewpoint.

Fig. 8 plots the temperature evolution in both cyclic and quasi-cyclic models. The plots are presented for a time interval from the beginning of the evolution to the point where the trajectory on the energy phase plane intersects the abscissa axis. It is done, as has been already discussed, because in such a model the temperature becomes negative after the phase curve intersects the axis $\dot{W} = 0$. It indicates a thermal equilibrium violation near such points. The effective temperature of the field becomes negative while that of the fluid must remain non-negative.

7. CONCLUSION

We have constructed a cosmological model classification method on the basic of their energy phase curves. The method allows one, on a qualitative level, to determine the basic characteristics of models from the properties of their energy phase trajectories at the beginning and end points of the Universe evolution. Such characteristics are the positions of starting and end points as well as the slope of phase trajectories with respect to the axis $\dot{W} = 0$. The latter parameter determines the total evolution time of the Universe. As examples, we have analyzed some sufficiently extraordinary models and showed their features that can be established directly from the properties of the phase trajectories at their starting and end points. In particular, for cyclic models with two matter components, the field and the fluid, it has been shown that near the point where the phase trajectory crosses the axis $\dot{W} = 0$, a thermal equilibrium between the fluid and field fluctuations should be violated. Using the suggested approach, one can build cosmological models with prescribed thermodynamic and asymptotic properties. Although this paper considered only models with a thermodynamic equilibrium between the field and fluid components, the approach is also suitable for building models with violated thermal equilibrium.

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